Name ______________ Date ______________________________

Proportions Proverb

Solve each proportion. Place the equation letter above the answer. The statement is a paraphrase of a proverb.

1. $\frac{3}{4} = \frac{12}{A}$
2. $\frac{4}{5} = \frac{C}{15}$
3. $\frac{5}{D} = \frac{20}{10}$
4. $\frac{E}{21} = \frac{2}{6}$
5. $\frac{F}{4} = \frac{7}{8}$
6. $\frac{3}{I} = \frac{30}{7}$
7. $\frac{15}{L} = \frac{20}{L + 0.5}$
8. $\frac{M}{17} = \frac{2}{4}$
9. $\frac{4}{2.8} = \frac{3}{N}$
10. $\frac{P}{3} = \frac{1}{15}$
11. $\frac{Q}{18} = \frac{5}{4}$
12. $\frac{7}{2} = \frac{R}{R - 15}$
13. $\frac{5}{S} = \frac{4}{S - 2.5}$
14. $\frac{18}{60} = \frac{T}{5T - 1.5}$
15. If four cans of stew cost $5.00, what will be the unit (U) cost of 1 can?
16. Jamie rode her bike 52.5 miles east in 3 hours. What was her velocity (V) per hour?
17. Tony ran for a total of 75 yards during 6 games. At this rate, how many yards (Y) will Tony run in the remaining 2 games?

__ O __ __   __ __ __ __ __   __ O __ __   __ __ __ __ __
2.5 2.1 0.9   0.2 1.5 16 12 7   25 1.25 21   8.5 7 16 2.1 12.5

O __   __ O __ __ __ __ __ __ __ __   __ __
3.5   12 2.1 17.5 7 25 16 2.1 12 7   0.7 2.1

__ . __ O __ __   O __   __ O __ __   __ __ __ __ __ __
3.5 21 2.1 0.9   3.5   25 1.25 21   7 22.5 1.25 0.7 2.1 7

__ __ __ __ __ __ .
16 2.1 0.7 8.5 16 1.5

Write the familiar proverb.

_______ _______ _______ _______ _______ _______ _______ .

Name ____________________ Date ____________

Manipulating Powers

14. $(x^{-2} \cdot y)^{-3}$

15. $\dfrac{12x^5}{3x^7}$

16. $\dfrac{8d}{(10d^{-4})(9d^2)}$

17. $^{-}2x^{-2}$

18. $x^{1/3} \cdot x^{2/3}$

19. $\left(\dfrac{8x}{125}\right)^{-2}$

20. $\dfrac{a^4 \cdot b^6 \cdot a^9}{b^{-2}}$

21. $\dfrac{x^{-4}y^{-6}}{x^2y^5z}$

22. $\left(\dfrac{x^2}{(xz)^2}\right)^{-2}$

23. $\left(\dfrac{x^2y^1z}{a^4b^{-7}}\right)^{-3}$

24. $(x^2y^2)^{-2} \cdot x^4y^{19}$

25. $\left(\dfrac{x^{-4}}{y^6}\right)^3 \cdot \left(\dfrac{x}{y^{-4}}\right)^{-4}$

26. $(a^2b^1c^8)^6 \cdot a^{-9} \cdot b^4 \cdot x$

27. $\left(\dfrac{x^{-4}b^{-1}}{4}\right)^{-3} \cdot 2x^5$

28. $(a^9b^{-2}c^1)^{-4} \cdot \left(\dfrac{ab}{x}\right)^3$

29. $\left(\dfrac{x^{-4}y^{-6}z^{10}}{a^1b^2c^{-4}}\right)^{-2} \cdot \left(\dfrac{a^1bc^{-4}}{x^6yz^9}\right)^5$

Name ______________________ Date ______________________

Manipulating Powers

1) $(a^x)^y = a^{xy}$	**4)** $(ab)^x = a^x b^x$	**7)** $\frac{1}{a^{-x}} = a^x$
2) $a^x \cdot a^y = a^{x+y}$	**5)** $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$	
3) $\frac{a^x}{a^y} = a^{x-y}$	**6)** $a^{-x} = \frac{1}{a^x}$	

Simplify each expression.

Example: $(x^2)^4 = x^{2 \cdot 4} = x^8$

1. $x^4 \cdot x^2$

2. $\frac{x^8}{x^6}$

3. $(x^2 y)^3$

4. $\left(\frac{x}{y^3}\right)^5$

5. y^{-15}

6. $\frac{1}{x^{-15}}$

7. $\frac{a^6}{a^9}$

8. $(2c^2)^3$

9. $\frac{n^4 \cdot n^6}{n^8 \cdot n^2}$

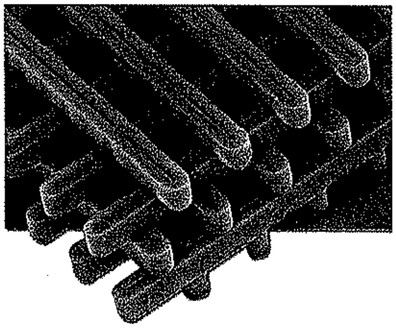
10. $4a^5 \cdot 3a^3$

11. $\left(\frac{v}{3}\right)^4 \cdot \left(\frac{5}{v}\right)^2$

12. $(x^{-2})^2$

13. $\left(\frac{2}{x}\right)^{-1}$

Scientific Notation



Scientists use photonic lattices to trap or bend light within extremely tiny spaces. The micrograph above shows a photonic lattice made of silicon rods. Each rod is 1.2 microns, or 1.2×10^{-6} meter, wide.

Very large and very small numbers are often expressed in scientific notation (also known as exponential form). In scientific notation, a number is written as the product of two numbers: a coefficient, and 10 raised to a power. For example, the number 84,000 written in scientific notation is 8.4×10^4. The coefficient in this number is 8.4. In scientific notation, the coefficient is always a number greater than or equal to one and less than ten. The power of ten, or exponent, in this example is 4. The exponent indicates how many times the coefficient 8.4 must be multiplied by 10 to equal the number 84,000.

$$\underbrace{8.4 \times 10^4}_{\text{exponential form (scientific notation)}} = 8.4 \times 10 \times 10 \times 10 \times 10 = \underbrace{84{,}000}_{\text{standard form}}$$

When writing numbers greater than ten in scientific notation, the exponent is equal to the number of places that the decimal point has been moved to the left.

$$6{,}300{,}000 = 6.3 \times 10^6 \qquad 94{,}700 = 9.47 \times 10^4$$

6 places   4 places

Numbers less than one have a negative exponent when written in scientific notation. For example, the number 0.000 25 written in scientific notation is 2.5×10^{-4}. The negative exponent -4 indicates that the coefficient 2.5 must be divided four times by 10 to equal the number 0.000 25, as shown below.

$$\underbrace{2.5 \times 10^{-4}}_{\text{exponential form (scientific notation)}} = \frac{2.5}{10 \times 10 \times 10 \times 10} = \underbrace{0.000\,25}_{\text{standard form}}$$

When writing numbers less than one in scientific notation, the value of the exponent equals the number of places the decimal has been moved to the right. The sign of the exponent is negative.

$$0.000\,008 = 8 \times 10^{-6} \qquad 0.00736 = 7.36 \times 10^{-3}$$

6 places   3 places

If your calculator has an exponent key, you can enter numbers in scientific notation when doing calculations. See the section on using a calculator (pages R62–R65) for more information on calculator operations that involve scientific notation.

Multiplication and Division

To multiply numbers written in scientific notation, multiply the coefficients and add the exponents.

$(3 \times 10^4) \times (2 \times 10^2) = (3 \times 2) \times 10^{4+2} = 6 \times 10^6$
$(2.1 \times 10^3) \times (4.0 \times 10^{-7}) = (2.1 \times 4.0) \times 10^{3+(-7)} = 8.4 \times 10^{-4}$

To divide numbers written in scientific notation, divide the coefficients and subtract the exponent in the denominator from the exponent in the numerator.

$$\frac{3.0 \times 10^5}{6.0 \times 10^2} = \left(\frac{3.0}{6.0}\right) \times 10^{5-2} = 0.5 \times 10^3 = 5.0 \times 10^2$$

Addition and Subtraction

If you want to add or subtract numbers expressed in scientific notation and you are not using a calculator, then the exponents must be the same. For example, suppose you want to calculate the sum of 5.4×10^3 and 8.0×10^2. First, rewrite the second number so that the exponent is a 3.

$$8.0 \times 10^2 = 0.80 \times 10^3$$

Now add the numbers.

$$(5.4 \times 10^3) + (0.80 \times 10^3) = (5.4 + 0.80) \times 10^3 = 6.2 \times 10^3$$

Follow the same rule when you subtract numbers expressed in scientific notation without the aid of a calculator.

$$\begin{aligned}(3.42 \times 10^{-5}) - (2.5 \times 10^{-6}) &= (3.42 \times 10^{-5}) - (0.25 \times 10^{-5}) \\ &= (3.42 - 0.25) \times 10^{-5} = 3.17 \times 10^{-5}\end{aligned}$$

SAMPLE PROBLEM MH-1

Using Scientific Notation in Arithmetic Operations

Solve each problem, and express your answer in correct scientific notation.

a. $(8.0 \times 10^{-2}) \times (7.0 \times 10^{-5})$
b. $(7.1 \times 10^{-2}) + (5 \times 10^{-3})$

Solution

Follow the rules described above for multiplying and adding numbers expressed in scientific notation.

a. $(8.0 \times 10^{-2}) \times (7.0 \times 10^{-5}) = (8.0 \times 7.0) \times 10^{-2+(-5)}$
$= 56 \times 10^{-7} = 5.6 \times 10^{-6}$

b. $(7.1 \times 10^{-2}) + (5 \times 10^{-3}) = (7.1 \times 10^{-2}) + (0.5 \times 10^{-2})$
$= (7.1 + 0.5) \times 10^{-2} = 7.6 \times 10^{-2}$

Practice the Math

1. Express each number in scientific notation
   a. 500,000 b. 285.2
   c. 0.000 000 042 d. 0.0002
   e. 0.030 06 f. 83,700,000
2. Write each number in standard form.
   a. 4×10^{-3} b. 3.4×10^{5}
   c. 0.045×10^{4} d. 5.9×10^{-6}
3. Solve each problem and express your answer in scientific notation.
   a. $(2 \times 10^{9}) \times (4 \times 10^{3})$
   b. $(6.2 \times 10^{-3}) \times (1.5 \times 10^{1})$
   c. $(10^{-4}) \times (10^{8}) \times (10^{-2})$
   d. $(3.4 \times 10^{-3}) \times (2.5 \times 10^{-5})$
4. Solve each problem and express your answer in scientific notation.
   a. $(9.4 \times 10^{-2}) - (2.1 \times 10^{-2})$
   b. $(6.6 \times 10^{-8}) + (5.0 \times 10^{-9})$
   c. $(6.7 \times 10^{-2}) - (3.0 \times 10^{-3})$
5. Solve each problem and express your answer in scientific notation.
   a. $\dfrac{(3.8 \times 10^{-3}) \times (1.2 \times 10^{6})}{8 \times 10^{4}}$
   b. $(1.4 \times 10^{2}) \times (2 \times 10^{8}) \times (7.5 \times 10^{-4})$
   c. $\dfrac{6.6 \times 10^{6}}{(8.8 \times 10^{-2}) \times (2.5 \times 10^{3})}$
   d. $\dfrac{(1.2 \times 10^{-3})^{2}}{(10^{-2})^{3} \times (2.0 \times 10^{-3})}$
6. Express each measurement in scientific notation.
   a. The length of a football field: 91.4 m.
   b. The diameter of a carbon atom: 0.000 000 000 154 m.
   c. The diameter of a human hair: 0.000 008 m.
   d. The average distance between the centers of the sun and Earth: 149,600,000,000 m.

Applying Scientific Notation to Chemistry

7. The following expressions are solutions to typical chemistry problems. Calculate the answer for each expression. Make sure to cancel units as you write your solutions.
   a. $5.6 \times 10^{3}\ \text{mm} \times \left(\dfrac{1\ \text{m}}{10^{3}\ \text{mm}}\right) \times \left(\dfrac{10^{9}\ \text{nm}}{1\ \text{m}}\right) = ?$
   b. $6.8 \times 10^{4}\ \text{cg H}_2\text{O} \times \left(\dfrac{1\ \text{g H}_2\text{O}}{10^{2}\ \text{cg H}_2\text{O}}\right) \times \left(\dfrac{1\ \text{mL H}_2\text{O}}{1\ \text{g H}_2\text{O}}\right) \times \left(\dfrac{1\ \text{L H}_2\text{O}}{10^{3}\ \text{mL H}_2\text{O}}\right) = ?$
   c. $4.0 \times 10^{2}\ \text{mL NaOH} \times \left(\dfrac{1\ \text{L NaOH}}{10^{3}\ \text{mL NaOH}}\right) \times \left(\dfrac{6.5 \times 10^{-2}\ \text{mol NaOH}}{1\ \text{L NaOH}}\right) = ?$
8. A cube of aluminum measures 1.50×10^{-2} m on each edge. Use the following expression to calculate the surface area of the cube.

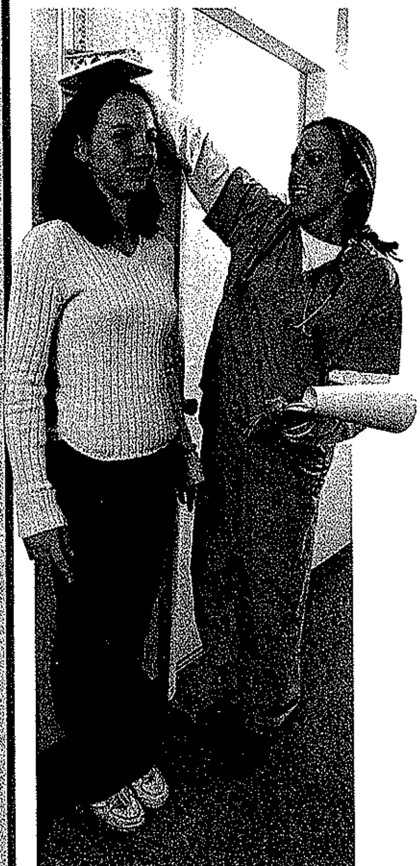
   $$\text{Surface area} = 6 \times (1.50 \times 10^{-2}\ \text{m})^{2} = ?$$

9. A small gold (Au) nugget has a mass of 3.40×10^{-3} kg. Use the following expression to calculate the number of gold atoms contained in the nugget.

   $$3.40 \times 10^{-3}\ \text{kg Au} \times \left(\frac{10^{3}\ \text{g Au}}{1\ \text{kg Au}}\right) \times \left(\frac{1\ \text{mol Au}}{197.0\ \text{g Au}}\right) \times \left(\frac{6.02 \times 10^{23}\ \text{atoms Au}}{1\ \text{mol Au}}\right) = ?$$

10. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^{3}$, where $\pi = 3.14$ and r = radius. What is the volume of a spherical drop of water with a radius of 2.40×10^{-3} m?

Conversion Problems and Dimensional Analysis



A person's height can be measured in units such as feet, inches, or meters. Conversion factors allow you to convert from one unit to another.

Many problems in both everyday life and in the sciences involve converting measurements. These problems may be simple conversions between the same kinds of measurement. For example:

a. A person is five and one-half feet tall. Express this height in inches.
b. A flask holds 0.575 L of water. How many milliliters of water is this?

In other cases, you may need to convert between different kinds of measurements.

c. How many gallons of gasoline can you buy for $15.00 if gasoline costs $1.42/gallon?
d. What is the mass of 254 cm^3 of gold if the density of gold is 19.3 g/cm^3?

More complex conversion problems may require conversions between measurements expressed as ratios of units. Consider the following examples.

e. A car is traveling at 65 miles/hour. What is the speed of the car expressed in feet/second?
f. The density of nitrogen gas is 1.17 g/L. What is the density of nitrogen expressed in micrograms/deciliter (μg/dL)?

Problems **a** through **f** can be solved using a method that is known by a few different names—dimensional analysis, factor label, and unit conversion. These names emphasize the fact that the dimensions, labels, or units of the measurements in a problem—the units in the given measurement(s) as well as the units desired in the answer—can help you write the solution to the problem.

Dimensional analysis makes use of ratios called conversion factors. A conversion factor is a ratio of two quantities equal to one another. For example, to work out problem **a**, you must know the relationship 1 ft = 12 in. The two conversion factors derived from this equality are shown below.

$$\frac{1\text{ ft}}{12\text{ in}} = 1\text{ (unity)} = \frac{12\text{ in}}{1\text{ in}}$$

To solve problem **a** by dimensional analysis, you must multiply the given measurement (5.5 ft) by a conversion factor that allows the *feet* units to cancel, leaving the unit *inches*—the unit of the requested answer.

$$5.5\ \cancel{\text{ft}} \times \frac{12\text{ in}}{1\ \cancel{\text{ft}}} = 66\text{ in}$$

Carefully study the solutions to the remaining five example problems below. Notice that in each solution, the conversion factors are written so that the unit of the given measurement cancels, leaving the correct unit for each answer. When working conversion problems, the equalities needed to write the conversion factor may be given in the problem. This is true in examples **c** and **d**. In other problems, you need to either know or look up the necessary equalities, as in examples **b**, **e**, and **f**.

b. $0.575\ \cancel{\text{L}} \times \dfrac{10^3\ \text{mL}}{1\ \cancel{\text{L}}} = 575\ \text{mL}$

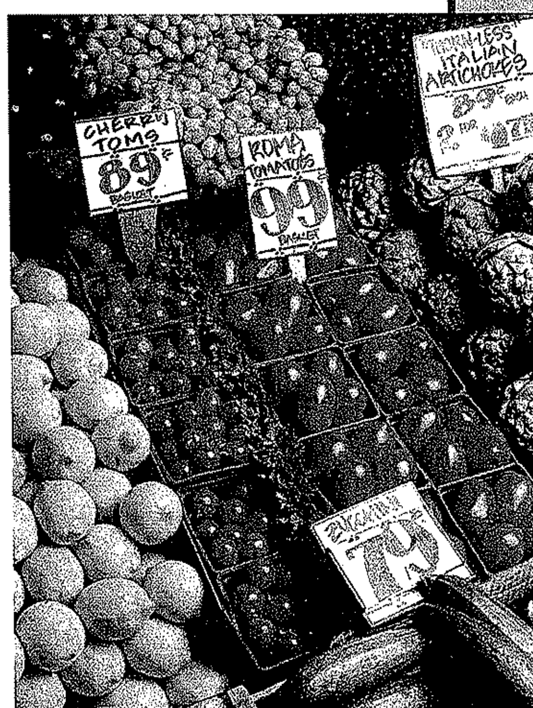
c. $\$15.00 \times \dfrac{1\ \text{gal}}{\$1.42} = 10.6\ \text{gal}$

d. $254\ \cancel{\text{cm}^3} \times \dfrac{19.3\ \text{g}}{1\ \cancel{\text{cm}^3}} = 4.90 \times 10^3\ \text{g}$

e. $\dfrac{65\ \cancel{\text{mi}}}{1\ \cancel{\text{h}}} \times \dfrac{5280\ \text{ft}}{1\ \cancel{\text{mi}}} \times \dfrac{1\ \cancel{\text{h}}}{3600\ \text{s}} = 95\ \text{ft/s}$

f. $\dfrac{1.17\ \cancel{\text{g}}}{1\ \cancel{\text{L}}} \times \dfrac{10^6\ \mu\text{g}}{1\ \cancel{\text{g}}} \times \dfrac{1\ \cancel{\text{L}}}{10\ \text{dL}} = 1.17 \times 10^5\ \mu\text{g/dL}$

Based on the prices advertised below, you can derive conversion factors that relate cost to a certain amount of produce. For example, you can write $1.75/2 Italian artichokes or $0.79/zucchini.



SAMPLE PROBLEM MH-5

Applying Dimensional Analysis

A grocer is selling oranges at "3 for $1." How much would it cost to buy a dozen oranges?

Solution

The following equality is given in the problem.

$$3\ \text{oranges} = \$1$$

You can write two conversion factors based on this relationship.

$$\frac{\$1}{3\ \text{oranges}} \quad \text{and} \quad \frac{3\ \text{oranges}}{\$1}$$

The given unit is oranges; the desired unit is dollars. Thus, use the conversion factor on the left to convert from oranges to dollars. One dozen equals 12, so you can start the calculation with the measurement 12 oranges.

$$12\ \cancel{\text{oranges}} \times \frac{\$1}{3\ \cancel{\text{oranges}}} = \$4$$

The given unit (oranges) cancels, leaving the desired unit (dollars) in the answer.

Practice the Math

Use the following equalities for Questions 1–3.

60 s = 1 min	5.50 yd = 1 rod
12 in = 1 ft	7 days = 1 wk
60 min = 1 h	5280 ft = 1 mi
3 ft = 1 yd	365 days = 1 yr
24 h = 1 day	

1. Write the conversion factor need for each unit conversion.
   a. feet $\rightarrow$ yards
   b. years $\rightarrow$ days
   c. yards $\rightarrow$ rods
   d. days $\rightarrow$ hours
   e. feet $\rightarrow$ miles
   f. seconds $\rightarrow$ minutes
2. Solve each problem by dimensional analysis.
   a. How many feet long is the 440-yard dash?
   b. Calculate the number of minutes in two weeks.
   c. Calculate the number of days in 1800 h.
   d. How many miles is 660 ft?
   e. How many inches long is a 100-yd football field?
   f. Calculate the number of hours in one year.
   g. How many rods are in 12 miles?
   h. Calculate the number of minutes in 7 days.
3. Solve each problem by dimensional analysis.
   a. A student walks at a brisk 3.50 mi/h. Calculate the student's speed in yards/minute.
   b. Water runs through a hose at the rate of 2.5 gal/min. What is the rate of water flow in units gallons/day?
   c. A clock gains 2.60 s each hour (2.6 s gained/h). What is the rate of time gained in minutes/week?
   d. A spider travels 115 inches in 1 min (speed = 115 in/min). What is the speed of the spider in miles/hour?

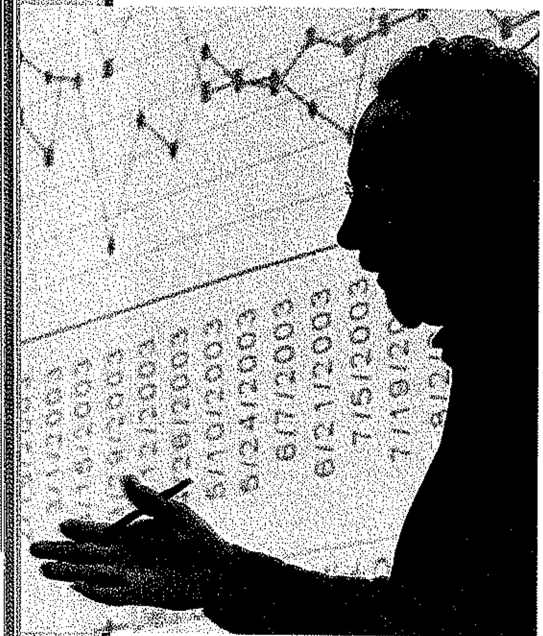
Applying Dimensional Analysis to Chemistry

Use the following metric relationships to work out Questions 4 and 5.

10^3 m = 1 km	10^9 nm = 1 m
10 dm = 1 m	10^{12} pm = 1 m
10^2 cm = 1 m	10^3 cm^3 = 1 L
10^3 mm = 1 m	1 mL = 1 cm^3
10^6 μm = 1 m	1 g H_2O = 1 mL H_2O

4. Perform the following conversions.
   a. 45 m to kilometers
   b. 4×10^7 nm to meters
   c. 8.5 dm to millimeters
   d. 8.2×10^{-4} μm to centimeters
   e. 0.23 km to decimeters
   f. 865 cm^3 to liters
   g. 7.28×10^2 pm to micrometers
   h. 56 g H_2O to L H_2O
5. Perform the following conversions.
   a. 4.5 m/s to millimeters/minute
   b. 7.9×10^{-2} km/h to decimeters/minute
   c. 77 mL H_2O/s to liters H_2O/hour
   d. 3.34×10^4 nm/min to centimeters/second

Graphing



A line graph can be used to show the relationship between two variables. The manipulated variable is plotted on the x-axis. The responding variable is plotted on the y-axis.

The relationship between two variables in an experiment is often determined by graphing the experimental data. A graph is a "picture" of the data. Once a graph is constructed, additional information can be derived about the variables.

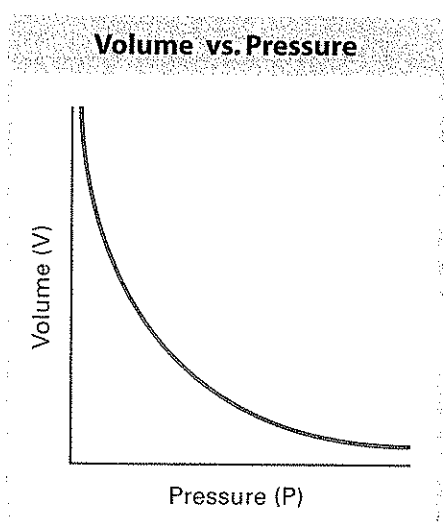
In constructing a graph, you must first label the axes. The manipulated variable (also known as the independent variable) is plotted on the x-axis. This is the horizontal axis. The manipulated variable is controlled by the experimenter. When the independent variable is changed, a corresponding change in the responding variable (also known as the dependent variable) is measured. The responding variable is plotted on the y-axis. This is the vertical axis. The label on each axis should include the unit of the quantity being graphed.

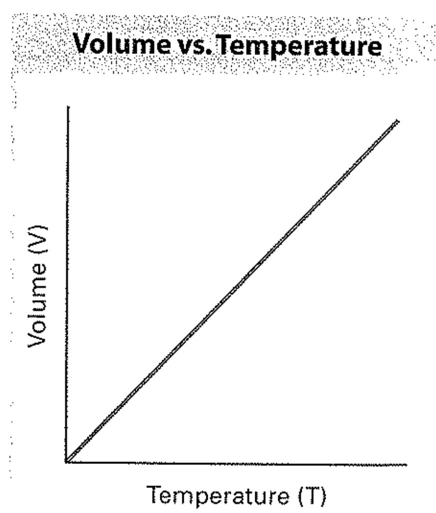
Before data can be plotted on a graph, each axis must be scaled. Each interval on the scale must represent the same amount. To make it easy to find numbers along the scale, the interval chosen is usually a multiple of 1, 2, 5, or 10. Although each scale can start at zero, this is not always practical.

Data are plotted by putting a point at the intersection of corresponding values of each pair of measurements. Once the data has been plotted, the points are connected by a smooth curve. A smooth curve comes as close as possible to all the plotted points. It may in fact not touch any of them.

Inverse and Direct Proportionalities

Depending on the relationship between two variables, a plotted curve may or may not be a straight line. Two common curves are shown below.





The volume- vs. -pressure curve is typical of an inverse proportionality. As the manipulated variable (P) increases, the responding variable (V) decreases. The product of the two variables at any point on the curve of an inverse proportionality is a constant. Thus, $V \times P =$ constant.

The straight line in the volume- vs. -temperature graph is typical of a direct proportionality. As the manipulated variable (T) increases, there is a corresponding increase in the responding variable (V). A straight line can be represented by the following general equation.

$$y = mx + b$$

The variables y and x are plotted on the vertical and horizontal axes, respectively. The y-intercept, b, is the value of y when x is zero. The slope, m, is the ratio of the change in y (Δy) for a corresponding change in x (Δx).

$$m = \frac{\Delta y}{\Delta x}$$

Plotting and Interpreting Graphs

Consider the following set of data about a bicyclist's trip. Assume that the bicyclist rode at a constant speed.

Distance from home (km)	15	25	35	50	75
Time (h)	1	2	3	4.5	7

Graph these data using time as the manipulated variable, and then use the graph to answer the following questions.

a. How far from home was the bicyclist at the start of the trip?

b. How long did it take for the bicyclist to get 40 km from home?

The plotted points are shown in the graph below. Each point was plotted by finding the value of time on the x-axis, then moving up vertically to the value of the other variable (distance). A smooth curve (in this case a straight line), has been drawn through the points.

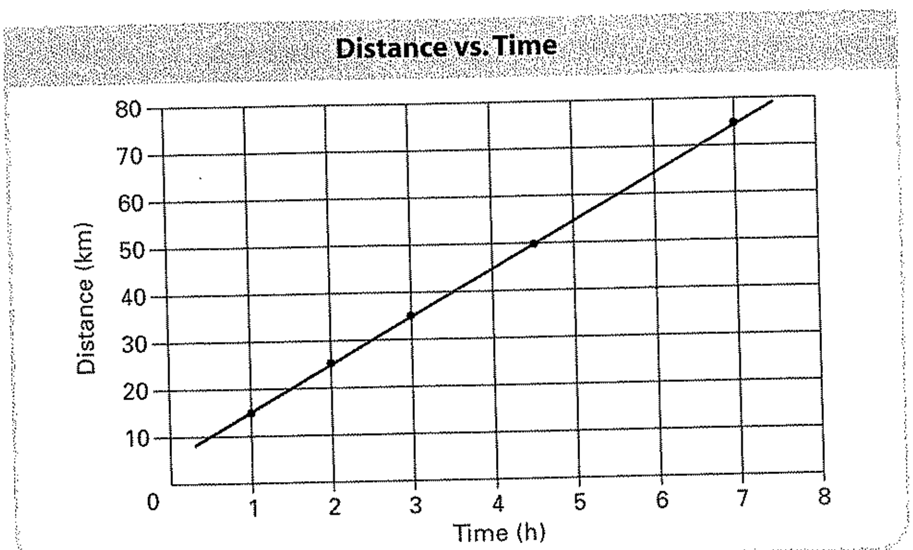


Figure 1

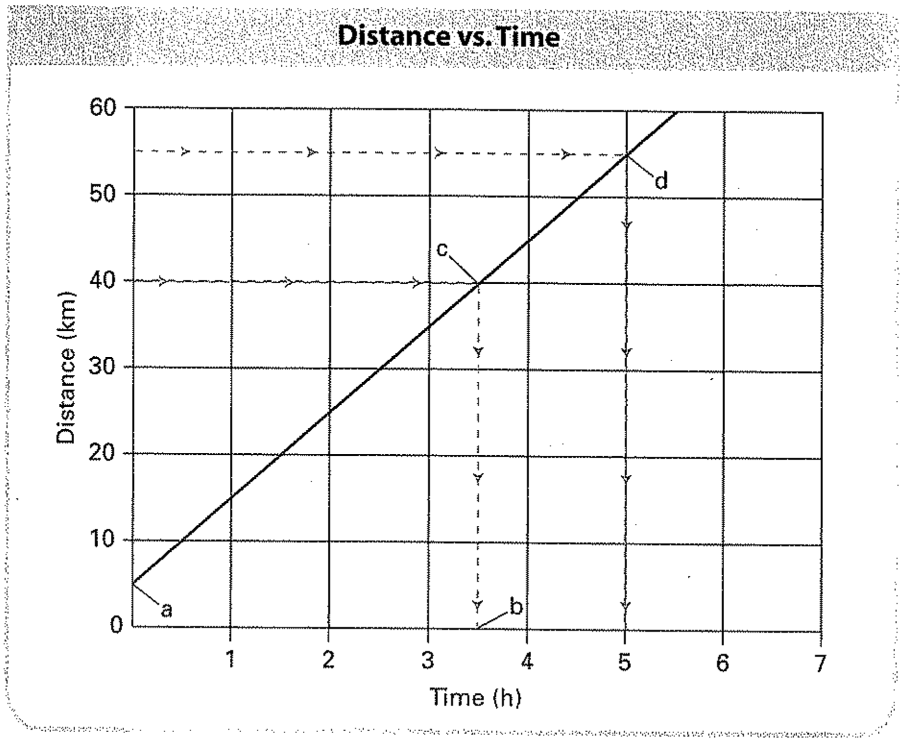


Figure 2

To answer problem **a**, extend the curve so that it intersects the y-axis, as in the graph above. The graph shows that the bicyclist started the trip 5 km from home. This is the value of the vertical axis (distance) when the time elapsed is zero (point *a* on the graph).

For problem **b**, find the value 40 km on the y-axis of the graph. Move to the right (horizontally) in the graph until you reach the line. Drop down vertically and read the value of time at this point (point *b*). It takes the bicyclist 3.5 h to get 40 km from home.

SAMPLE PROBLEM MH-8

Interpreting Graphs

Use the distance- vs. -time graph above to calculate the bicyclist's average speed in kilometers per hour (km/h).

Solution

Speed is distance/time. The average speed of the bicyclist is the slope of the line in the graph. Calculate the slope using the values for time and distance corresponding to points *c* and *d*.

$$m = \frac{\Delta y}{\Delta x} = \frac{55\text{ km} - 40\text{ km}}{5\text{ h} - 3.5\text{ h}} = \frac{15\text{ km}}{1.5\text{ h}} = 10\text{ km/h}$$

You can now write an equation for the line.

$$y = mx + b$$

$$\text{Distance} = (10\text{ km/h})(\text{time}) + 5\text{ km}$$

Practice the Math

1. A bicyclist wants to ride 100 kilometers. The data below show the time required to ride 100 kilometers at different average speeds.

Time (h)	4	5	8	10	15	20
Avg speed (km/h)	25	20	12.5	10	6.7	5

   a. Graph the data, using average speed as the independent variable.
   b. Is this a direct or inverse proportionality?
   c. What average speed must be maintained to complete the ride in 12 hours?
   d. If a bicyclist's average speed is 18 km/h, how long does it take to ride 100 km?

2. The data below shows how the mass of a baby varies with its age during its first year of life.

Age(days)	40	110	200	270	330
Mass (kg)	4.0	5.4	7.3	8.6	9.9

   a. Graph the data, using age as the independent variable.
   b. Derive an equation in the form of $y = mx + b$. Include units on the values of y and b.
   c. Why would the values of y and b be of interest to both the baby's parents and physician?

Applying Graphs to Chemistry

3. Use the following data to draw a graph that shows the relationship between the Fahrenheit and Celsius temperature scales. Make °F the responding variable. Use the graph to derive an equation relating °F and °C. Then use the graph or the equation to find values for y_1, x_2, and x_3.

Temperature (°F)	50	212	356	−4	y_1	70	400
Temperature (°C)	10	100	180	−20	70	x_2	x_3

4. Different volumes of the same liquid were added to a graduated cylinder sitting on a balance. After each addition of liquid the total volume of liquid and mass of the liquid-filled graduated cylinder was recorded in the table below.

Total volume of liquid (mL)	10	25	45	70	95
Mass of liquid and cylinder (g)	138	159	187	222	257

   a. Graph the data, using volume as the manipulated variable.
   b. What is the y-intercept of the line? Make sure to include the unit.
   c. What does the value of the y-intercept represent?
   d. Calculate the slope of the line, and make sure to include the unit.
   e. What does the slope of the line represent for this liquid?
   f. Write a general equation that represents the line in your graph.

5. A student collected the following data or a fixed volume of gas.

Temperature (°C)	10	20	40	70	100
Pressure (mm Hg)	726	750	800	880	960

   a. Graph the data, using pressure as the responding variable.
   b. Is this a direct or inverse proportionality?
   c. At what temperature is the gas pressure 822 mm Hg?
   d. What is the pressure of the gas at a temperature of 0°C?
   e. How does the pressure of the gas change with a change in temperature?
   f. Write an equation relating the pressure and temperature of the gas.

Algebraic Equations

Many relationships in chemistry can be expressed as simple algebraic equations. However, the equation given is not always in the form that is most useful in figuring out a particular problem. In such a case, you must first solve the equation for the unknown quantity; this is done by rearranging the equation so that the unknown is on one side of the equation, and all the known quantities are on the other side.

Solving Simple Equations

An equation is solved using the laws of equality. The laws of equality are summarized as follows: *If equals are added to, subtracted from, multiplied by, or divided by equals, the results are equal.* In other words, you can perform any of these mathematic operations on an equation and not destroy the equality, *as long as you do the same thing to both sides of the equation.* The laws of equality apply to any legitimate mathematic operation, including squaring, taking square roots, and taking the logarithm.

Consider the following equation, which states the relationship between the Kelvin and Celsius temperature scales.

$$K = {}^\circ C + 273$$

Can this equation be used to find the Celsius-temperature equivalent of 400 K? Yes, it can, if the equation is first solved for the unknown quantity, °C.

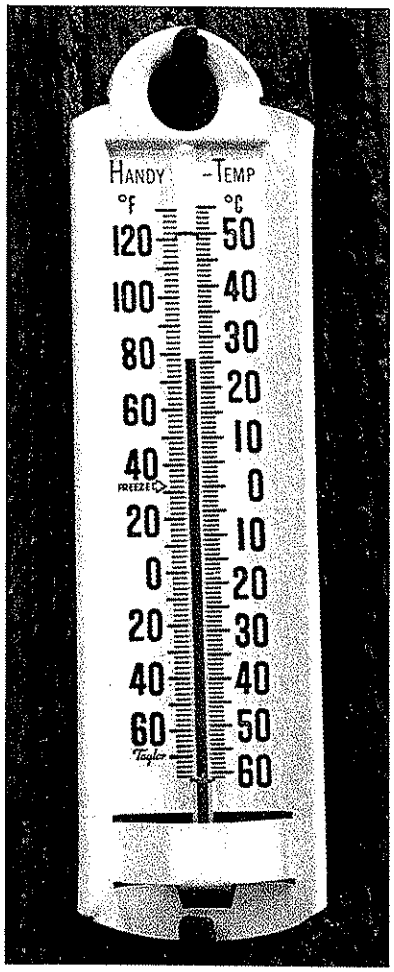
In the above example, to solve for °C, subtract 273 from both sides of the equation.

$$\begin{aligned} K &= {}^\circ C + 273 \\ K - 273 &= {}^\circ C + 273 - 273 \\ {}^\circ C &= K - 273 \end{aligned}$$

Now you have solved the equation for the unknown quantity. To calculate its value, substitute the known quantity into the solved equation.

$$\begin{aligned} {}^\circ C &= K - 273 \\ &= 400 - 273 \\ &= 127^\circ C \end{aligned}$$

A technician in a genetic research lab removes frozen cell samples from cryostorage. The cells are stored in liquid nitrogen at −196°C. What is this temperature in kelvins?



This thermometer shows both the Fahrenheit and Celsius temperature scales. What temperature in Celsius corresponds to −40°F? Verify the conversion by performing a calculation.

Another commonly used temperature scale is the Fahrenheit scale. The relationship between the Fahrenheit and Celsius temperature scales is given by the following equation.

$$°F = (1.8 \times °C) + 32$$

Suppose you want to use this equation to convert 365°F into degrees Celsius. To solve for °C, you must isolate it on one side of the equation. Since the right side of the equation has 32 added to the quantity (1.8 × °C), first subtract 32 from both sides of the equation, and then divide each side by 1.8.

$$\begin{aligned} °F &= (1.8 \times °C) + 32 \\ °F - 32 &= (1.8 \times °C) + 32 - 32 \\ °F - 32 &= (1.8 \times °C) \\ \frac{°F - 32}{1.8} &= \frac{1.8 \times °C}{1.8} \\ \frac{(°F - 32)}{1.8} &= °C \end{aligned}$$

Now that you have solved the equation for the unknown quantity (°C), you can substitute the known quantity (365°F) into the equation and calculate the answer.

$$°C = \frac{(°F - 32)}{1.8} = \frac{(365 - 32)}{1.8} = \frac{333}{1.8} = 185°C$$

SAMPLE PROBLEM MH-6

Solving Algebraic Equations

The heat (q) absorbed by the water in a calorimeter can be calculated using the following relationship.

$$q = m \times C \times \Delta T$$

In this expression, m is the mass of the water; C is the specific heat of water (4.18 J/g·°C); and ΔT is the change in temperature. If 120 g of water absorb 3500 J of heat, by how much will the temperature of the water increase?

Solution

First solve the equation for the unknown quantity, ΔT.

$$\begin{aligned} q &= m \times C \times \Delta T \\ \frac{q}{m \times C} &= \frac{m \times C \times \Delta T}{m \times C} \\ \frac{q}{m \times C} &= \Delta T \end{aligned}$$

Now substitute the known values for q, m, and C.

$$\begin{aligned} \Delta T &= \frac{q}{m \times C} \\ &= \frac{3500\ \cancel{J}}{(120\ g \times 4.18\ \cancel{J}/g \cdot °C)} = 7.0°C \end{aligned}$$

Practice the Math

1. Solve each equation for z.
   a. $xy + z = 5$
   b. $\frac{z}{a-4} = t$
   c. $\frac{b}{d} = \frac{2a}{z}$
   d. $\sqrt{z} = 2b$
2. Solve each equation for a. Then calculate a value for a if $b = 4$, $c = 10$, and $d = 2$.
   a. $bd = ac$
   b. $a + b = cd$
   c. $c + b = \frac{a}{d}$
   d. $\frac{bd}{a} = c^2$
3. Solve each equation for h. Then calculate a value for h if $g = 12$, $k = 0.4$, and $m = 1.5$.
   a. $kh = \frac{g}{m}$
   b. $\frac{(g - m)}{h} = k$
   c. $gh - k = m$
   d. $\frac{mk}{(g + h)} = 2$

Applying Algebra to Chemistry

4. Solve for v in the following equation.

$$d = \frac{m}{v}$$

   Let d = density, m = mass, and v = volume. What is the volume of 642 g of gold if the density of gold is 19.3 g/cm^3?
5. Solve for n in the following equation.

$$P \times V = n \times R \times T$$

   How many moles (n) of helium gas fill a 6.45-L (V) balloon at a pressure (P) of 105 kPa and a temperature (T) of 278 K?
   (R = 8.31 (L·kPa)/(K·mol))
6. Solve for V_2 in the following equation.

$$\frac{P_1 \times V_1}{T_1} = \frac{P_2 \times V_2}{T_2}$$

   A 2.50-L (V_1) sample of nitrogen gas at a temperature (T_1) of 308 K has a pressure (P_1) of 1.15 atm. What is the new volume (V_2) of the gas if the pressure (P_2) is increased to 1.80 atm and the temperature (T_2) decreased to 286 K?
7. Solve for P_{He} in the following equation.

$$P_{total} = P_{Ar} + P_{He} + P_{Kr}$$

   A mixture of gases has a total pressure (P_{total}) of 376 kPa. What is the partial pressure of helium (P_{He}), if P_{Ar} = 92 kPa and P_{Kr} = 144 kPa?
8. Solve for T_2 in the following equation.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

   What is the value of T_2 when V_1 = 5.0 L, V_2 = 15 L, and T_1 = 200 K?