Calculus BC Plus Summer Packet

Welcome to BC Plus!

In order to complete the curriculum before the AP Calculus BC Exam in May, it is necessary to do some preparatory work this summer. As you already know, Chapter 1 in your Calculus book is a general review of Algebra 2 and Pre-Calculus concepts.

Attached to this letter is a table of contents, the summer assignment, and the answers. The summer assignment, or review packet, helps you to focus on the mathematical skills and content you will need to use in solving Calculus problems. These problems deal with skills and content that you studied in AP Calculus AB. Use your AB Calculus notes to help you solve the review problems.

You are responsible for completing this summer assignment. The review packet must be completed and evidence of your understanding is to be shown on the packet with answers placed in the spaces where provided. Complete work must be shown to justify your answer, graphs must be carefully drawn and labeled with key points, and ***attempts must be made for each problem***. **DO NOT USE A CALCULATOR unless a problem specifically indicates for you to do so.** *If you use a calculator, you must set up what you entered into the calculator and the answer the calculator produced for you on your paper.*

Be prepared to turn in your completed summer assignment on the FIRST DAY of class. We will spend the first few days going over the packet together as a class and then you will be tested on the material. The problem packet will be collected and thoroughly graded and count towards part of your first quarter grade. It is important to check your answers with those provided in this packet.

At this level, doing homework is more than just getting the problems done. The problems should be a learning experience. Take your time and make sure that you understand the concepts behind each problem. Seek out help to deal with problems and concepts you find challenging. I recommend that you try to meet with other BC Plus students in small groups this summer to help each other. We are all in this together!

There are 2 Calculus books left in the Westport Library over the summer to use as a reference if necessary. You will also be able to download this summer assignment online from a link on the Staples High School website.

We are looking forward to seeing you on the first day of school. Good luck with the assignment!

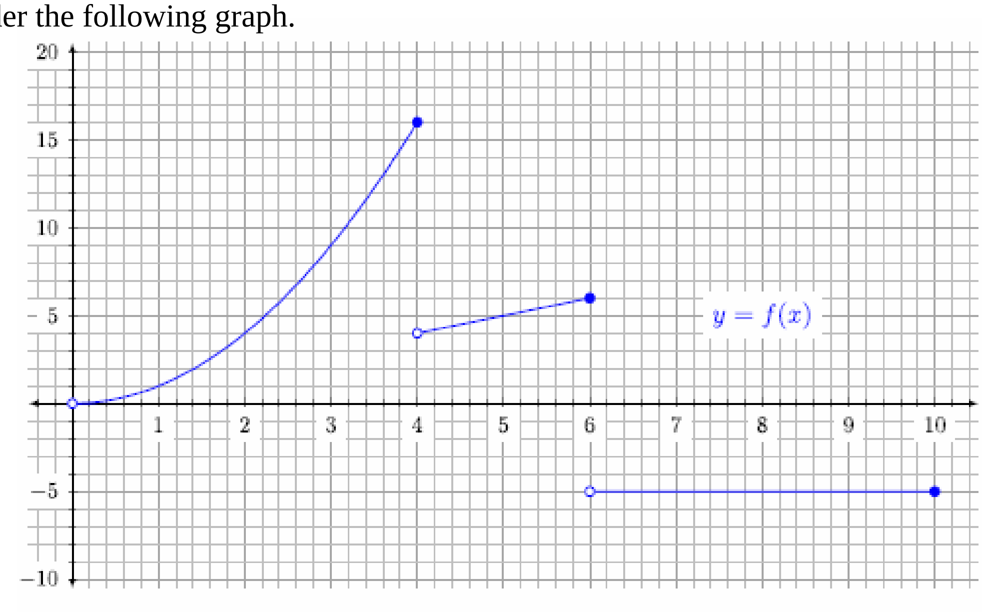
Mrs. Dana Dolyak
School email: ddolyak@westport.k12.ct.us

Mrs. Robin (Sacilotto) Hurlbut
School email: rhurlbut@westport.k12.ct.us

Chapter 2- Limits and Continuity

ALL PROBLEMS ARE NON CALCULATOR except #5 and #22 as indicated with

1) Consider the following graph.



1a) $\lim_{x \to 4^+} f(x)$ 1a) ____________

1b) $\lim_{x \to 4^-} f(x)$ 1b) ____________

1c) $\lim_{x \to 0^+} f(x)$ 1c) ____________

1d) $\lim_{x \to 0^-} f(x)$ 1d) ____________

1e) $\lim_{x \to 6^+} f(x)$ 1e) ____________

1f) $\lim_{x \to 6^-} f(x)$ 1f) ____________

2) Evaluate the following limits.

2a) $\lim_{t\to 3}\frac{t^2-4t+4}{t^2-4}$ 2a)____________

2b) $\lim_{x\to 0}\frac{3\sin 4x}{\sin 3x}$ 2b)____________

3) Assume that $\lim_{x\to 4} f(x)=2$ and $\lim_{x\to 4} g(x)=5$.

3a) $\lim_{x\to 4}\left(g(x)+1\right)$ 3a)____________

3b) $\lim_{x\to 4} x\,f(x)$ 3b)____________

3c) $\lim_{x\to 4} g^2(x)$ 3c)____________

3d) $\lim_{x\to 4}\frac{g(x)}{f(x)-3}$ 3d)____________

4) Evaluate the following limits for *x* approaching infinity.

4a) $\lim_{x\to\infty}\frac{x^4-5}{x^2+12}$ 4a)____________

4b) $\lim_{x\to\infty}\frac{x-5}{x^3}$ 4b)____________

4c) $\lim_{x\to\infty}\frac{21x^5+3x^3+15}{40x^5+91x^4+21x^4}$ 4c)____________

5. For $f(x) = \dfrac{x^7 + 9}{4x^6 - 4}$, evaluate the following by **graphing** $f(x)$ on a calculator.

5a) $\lim\limits_{x \to \infty} f(x)$ 5a)____________

5b) $\lim\limits_{x \to -\infty} f(x)$ 5b)____________

5c) $\lim\limits_{x \to -1^+} f(x)$ 5c)____________

5d) $\lim\limits_{x \to -1^-} f(x)$ 5d)____________

Evaluate the following limits.

6a) $\lim\limits_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$ 6a)____________

6b) $\lim\limits_{x \to 0} \dfrac{\tan x - x}{x^3}$ 6b)____________

7a) $\lim\limits_{x \to \infty} \dfrac{\ln(\ln x)}{x}$ 7a)____________

7b) $\lim\limits_{x \to \infty} \dfrac{e^x}{x^2}$ 7b)____________

Chapter 3 Derivatives

8) Take the derivative with respect to *x* of the following expressions. (Remember- no negative exponents in your answer)

8a) $x^3 + 2x^2 - \frac{x}{2} - 8$ 8a)________________________________

8b) $x^{-4/3} + 5x^{-1/2} + \ln x$ 8b)________________________________

8c) $\tan(4x)$ 8c)__________________

8d) $\sin x \cos x$ 8d)__________________________

8e) $\sin^2 x$ 8e)__________________________

8f) $\ln(2x+1)$ 8f)__________________________

8g) 2^x 8g)__________________________

8) Take the derivative with respect to *x* of the following expressions. (cont.)

8h) e^{2x} 8h)____________________

8i) $\tan^{-1}(5x)$ 8i)____________________

8j) $\cos x \csc x$ 8j)____________________

Solve the following.

9) $\dfrac{d}{dx}\int_{\pi}^{x}\left(t^{10}-7t^{8}\right)\left(\sin^{-1} t\right)dt$ 9)____________________________

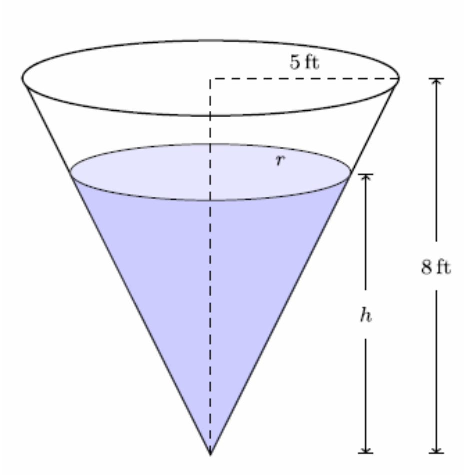
10) $\dfrac{d}{dx}\int_{3}^{x^2}\sqrt{1-t^4}\,dt$ 10)____________________________

11) $\dfrac{d}{dx}\int_{x}^{5x^3}e^{\cos t}\,dt$ 11)____________________________

12) Let *f* be the function defined by $f(x)=x^3+x$. If $g(x)=f^{-1}(x)$ and $g(2)=1$, what is the value of $g'(2)$? 12)____________

Chapter 4: Applications of Derivatives

13)



Water drains from a large, conical tank. The tank has a radius of 5 feet and a height of 8 feet. (The volume of a cone is given by $V = \dfrac{\pi r^2 h}{3}$) At what rate is the radius of the water in the tank changing when the water is draining from the tank at a rate of 1.5 ft^3/min and the height of the water in the tank, *h*, is 4 feet?

Chapter 5: The Definite Integral

t (seconds)	0	7	18	24	31	39
v(t) (meters per second)	2	4	–12	– 9	–5	6

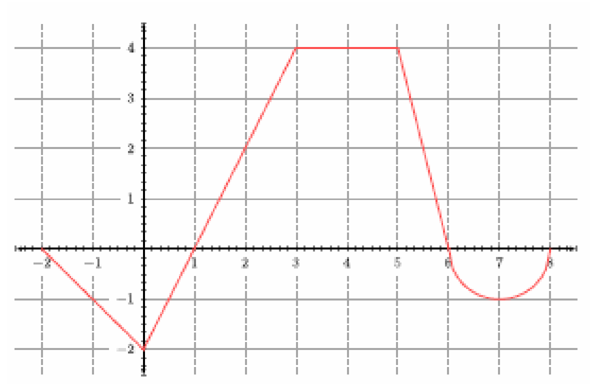
14) The velocity of a particle moving along the *x*-axis is modeled by a differentiable function *v*, where the position *x* is measured is meters, and time *t* is measured in seconds. Selected values of $v(t)$ are given in the table.

14a) Estimate the acceleration of the particle at $t = 35$ seconds. Show the computations the lead to your answer. Indicate units of measure.

14b) Using correct units, explain the meaning of $\frac{1}{21}\int_{18}^{39} v(t)\,dt$ in the context of this problem. Use a right Riemann sum with the three subintervals indicated by the data in the table to approximate $\frac{1}{21}\int_{18}^{39} v(t)\,dt$.

14c) For $0 \le t \le 39$, must there be a time(s) *t* where the particle must change direction indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

15) The figure shows the graph of the piecewise-linear function *f*, which consists of a triangle, a trapezoid, and a semi-circle. For $-2 \le x \le 8$, the function *g* is defined by $g(x) = \int_4^x f(t)\,dt$.



15a) Find $g(2)$, $g'(2)$, and $g''(5.2)$.

15b) Does *g* have a relative minimum, maximum, or neither at $x = 1$? Justify (explain) your answer.

15c) Does *g* have a point of inflection at $x = 6$? Justify (explain) your answer.

15d) Find the absolute maximum value of $g(x)$ on the interval $-2 \le x \le 8$. Justify (explain) your answer.

Chapter 6: Differential Equations and Mathematical Modeling

16) Consider the differential equation $\frac{dy}{dx} = (2 - y)\sin x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers. Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

17) Let $\dfrac{dy}{dx} = \dfrac{y+9}{3x^2 - 1}$.

17a) Find an equation for the tangent line to the curve at (2, 1).

17b) Find $\dfrac{d^2y}{dx^2}$ in terms of *x* and *y*.

17c) Is the curve concave up or down at (2, 1)? Explain your answer.

18) Evaluate each of the following expressions using geometry, known/basic anti-derivatives, u-substitution, integration by parts, and partial fraction decomposition.

18a) $\int e^x \cos(x)\,dx$ 18a)_________________________

18b) $\int x^2 \sin(x)\,dx$ 18b)_________________________

18c) $\int_{0}^{\pi/2} \sin x\, e^{\cos x}\, dx$

18c)_________________________

18d) $\int_{-2}^{2} \sqrt{4-x^2}\, dx$ (sketch a graph to help)

18d)_________________________

18e) $\int \frac{2x}{1-4x^2}\,dx$

18e)____________________________

18f) $\int \frac{4x\,dx}{3x^2+x-2}$

18f)____________________________

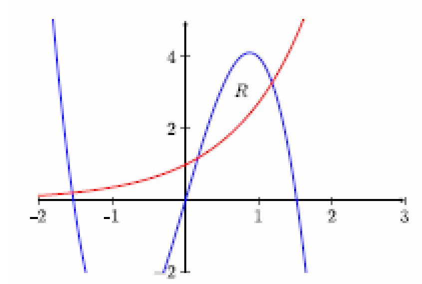
19) Given the equation $f(x) = 2x^3 - 5x + 2$, find the value(s) of *x* such that the average rate of change equals the instantaneous rate of change on $\left[-2,\ 1\right]$ and state the theorem (no abbreviations) that guarantees you can solve this problem.

20) Given that $f'(x) = \sin(2x)\,dx$ and $f\left(0\right) = 5$, find $f\left(\frac{\pi}{3}\right)$.

21) Consider the differential equation $\frac{dy}{dx} = x + y$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $y(1) = 0$. Use Euler's method with three steps of equal size to approximate $y(1.6)$.

Chapter 7: Applications of Definite Integrals

22) Let *R* be the region enclosed by $f(x)=7x-3x^3$ and $g(x)=e^{x-1/2}$ in the first quadrant. (Use a calculator for all parts)



22a) Find the area of *R*.

22b) The region *R* is rotated around the line $y=5$. Find the volume of the solid that is formed.

22c) Region *R* is the base of a solid. For this solid, each cross section perpendicular to the *x*-axis is an isosceles right triangle with a leg in *R*. Find the volume of the solid.

22d) The vertical line $x=k$ divides *R* into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of *k*.

ANSWERS

1a) 4

1b) 16

1c) 0

1d) DNE

1e) – 5

1f) 6

2a) $\frac{1}{5}$

2b) 4

3a) 6

3b) 8

3c) 25

3d) – 5

4a) ∞

4b) 0

4c) $\frac{21}{40}$

5a) ∞

5b) $-\infty$

5c) $-\infty$

5d) ∞

6a) $\frac{1}{2}$

6b) $\frac{1}{3}$

7a) 0

7b) ∞

8a) $3x^2+4x-\frac{1}{2}$

8b) $-\frac{4}{3x^{7/3}}-\frac{5}{2x^{3/2}}+\frac{1}{x}$

8c) $4\sec^2(4x)$

8d) $\cos^2 x-\sin^2 x$ or $\cos 2x$

8e) $2\sin x\cos x$ or $\sin 2x$

8f) $\frac{2}{2x+1}$

8g) $2^x\cdot\ln 2$

8h) $2e^{2x}$

8i) $\frac{5}{1+25x^2}$

8j) $-\cot^2 x-1$ or $-\csc^2 x$

9) $\left(x^{10}-7x^8\right)\sin^{-1} x$

10) $2x\sqrt{1-x^8}$

11) $15x^2e^{\cos(5x^3)}-e^{\cos x}$

12) $\frac{1}{4}$

13) $-\frac{3}{20\pi}$ ft/min

14a) $\frac{11}{8}$ m/sec^2

14b) average velocity,

$-\frac{41}{21}$ m/sec

14c) (7,18), (31, 39)

15a) $g(2)=7$, $g'(2)=2$,

$g''(5.2)=-4$

15b) relative min

15c) no POI

15d) abs max of 6 at $x = 6$

16) $y=2-e^{\cos x-1}$

17a) $y-1=\frac{10}{11}(x-2)$

17b) $\frac{d^2y}{dx^2}=\frac{(y+9)(1-6x)}{(3x^2-1)^2}$

17c) concave down

18a) $\frac{e^x\sin x+e^x\cos x}{2}+c$

18b)

$-x^2\cos x+2x\sin x+2\cos x+c$

18c) $e-1$

18d) 2π

18e) $-\frac{1}{4}\ln\left|1-4x^2\right|+c$

18f) $\frac{8}{15}\ln|3x-2|+\frac{4}{5}\ln|x+1|+c$

or $\ln\left|(3x-2)^{8/15}(x+1)^{4/5}\right|+c$

19) $x=-1,\ 1$, MVT

20) $\frac{23}{4}$

21) 0.856

22a) 2.187

22b) 35.670

22c) 1.093

22d) several possible answers