Name________________________

Period______

AP CALCULUS BC

Chapter 1 Syllabus / Summer Assignment

Congratulations! You made it to BC Calculus!

In order to complete the curriculum before the AP Exam in May, it is necessary to do some preparatory work this summer. As you will discover, Chapter 1 in your Calculus book is a general review of Algebra II and Pre-Calculus concepts.

Attached to this letter is an outline of the learning objectives, the summer assignment including a quick review, and the answers. The summer assignment, or review packet, helps you to focus on the mathematical skills and content you will need to use in solving Calculus problems. These problems deal with skills and content that you studied in Pre-Calculus. Use your Pre-Calculus notes to help you solve the review problems.

You are responsible for completing this summer assignment. The review packet must be completed and evidence of your understanding is to be shown on the packet with answers placed in the spaces where provided. Complete work must be shown to justify your answer, graphs must be carefully drawn and labeled, and attempts must be made for each problem. ** If a calculator has been used, then you must set up what you entered into the calculator and what the calculator produced for you on your paper.** **Be prepared to turn in your completed summer assignment on the first day of class- we will not be going over it together**. The problem packet will be collected and thoroughly graded and count towards part of your first quarter grade. It is important to check your answers with those provided in this packet.

At this level, doing homework is more than just getting the problems done. The problems should be a learning experience. Take your time and make sure you understand the concepts behind each problem. Seek out help to deal with problems and concepts you find challenging. I recommend that you try to meet with other AP Calculus BC students in small groups this summer to help each other. We are all in this together!

There are 2 Precalculus and 2 Calculus books left in the Westport Library over the summer to use as a reference if necessary. You will also be able to download this summer assignment online from a link on the Staples High School website.

I am looking forward to seeing you on the first day- good luck with the assignment!

Mrs. Robin (Sacilotto) Hurlbut
School email: *RHurlbut@westport.k12.ct.us*

SECTION 1.1 **LEARNING OBJECTIVES**

1. Be able to calculate the slope of a line
2. Be able to determine the equation of a line using slope-intercept form, point-slope form and the general form.
3. Know the relationship of slopes for parallel and perpendicular lines.
4. Be able to create a linear regression equation from data and use it to make predictions.

ASSIGNMENT Page 7: #37, 43, 51

SECTION 1.2 **LEARNING OBJECTIVES**

1. Know the definition of a function.
2. Be able to determine the domain and range of a function.
3. Be able to express answers in interval notation.
4. Be able to determine symmetry properties of a function.
5. Be able to identify odd and even functions.
6. Be able to graph piecewise functions.
7. Be able to compose functions.

ASSIGNMENT Page 17: #1, 7, 25, 33, 43, 49, 65

SECTION 1.3 **LEARNING OBJECTIVES**

1. Know the rules for operating with exponents.
2. Be able to create an exponential growth/decay equation.
3. Be able to solve exponential equations.

ASSIGNMENT Page 24: #9, 13, 25, 31

SECTION 1.4 **LEARNING OBJECTIVES**

1. Know how to graph a parametric equation, indicate initial and terminal points and direction in which it is traced (orientation).
2. Be able to convert a parametric equation into rectangular (Cartesian) form.
3. Be able to parametrize an equation.

ASSIGNMENT Page 30: #11, 19

SECTION 1.5 **LEARNING OBJECTIVES**

1. Be able to identify one-to-one functions.
2. Be able to determine the algebraic and graphical representation of a function and its inverse.
3. Be able to apply the properties of logarithms.

ASSIGNMENT Page 39: #7, 15, 33, 35, 37, 39, 41, 42

SECTION 1.6 **LEARNING OBJECTIVES**

1. Be able to convert between radians and degrees.
2. Be able to find arc length.
3. Be able to generate the graphs of the trigonometric functions.
4. Be able to identify periodicity and the even-odd properties of the trigonometric functions.
5. Be able to use the inverse trigonometric functions to solve problems.
6. Be able to identify period, amplitude, domain, range and various translations of the trigonometric functions.

ASSIGNMENT Page 48: #7, 15, 19, 25, 27, 29, 31

Forming Functions from Verbal Descriptions **LEARNING OBJECTIVES**

1. To form a function of one variable from a verbal description
2. Determine the minimum or maximum value of the function

ASSIGNMENT Page 161(in Precalculus book): #7, 9, 11, 19, 20, 23, 24, 25

SECTION 10.5 **LEARNING OBJECTIVES**

1. Be able to convert rectangular to polar and polar to rectangular equations
2. Be able to graph polar curves and identify certain known figures

ASSIGNMENT Page 558: #25, 28, 40, 46, 50, 57, 58, 71

AP CALCULUS BC SUMMER ASSIGNMENT Name:______________________

Period_______

SHOW EVIDENCE OF YOUR UNDERSTANDING ***for each problem*** *in this packet. This packet will be turned in on the first day of school and then graded. Every problem in this packet is to be completed* ***WITHOUT A CALCULATOR*** *unless indicated with this symbol:* 

Section 1.1

37. Find the value of *y* for which the line through *A* and *B* has the given slope *m*: $A(-2, 3)$, $B(4, y)$, $m = -2/3$

37.____________

43. For what value of *k* are the two lines $2x + ky = 3$ and $x + y = 1$ (a) parallel? (b) perpendicular?

43a)___________

43b)___________

51. Consider the circle of radius 5 centered at (0, 0). Find an equation of the line tangent to the circle at the point (3, 4) in slope intercept form.

51.________________________________

Section 1.2

1. Write a formula that expresses the first variable as a function of the second: 1.______________________________

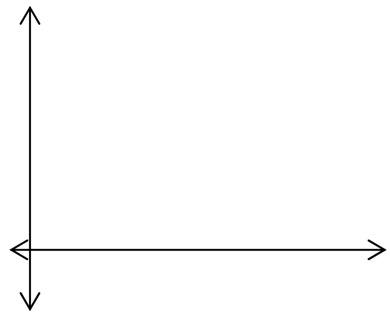
the area of a circle as a function of its diameter

7. Consider the function $y = 2 + \sqrt{x-1}$.

(a) State the domain:____________________________________

(b) State the range:_____________________________________

(c) Draw its graph.

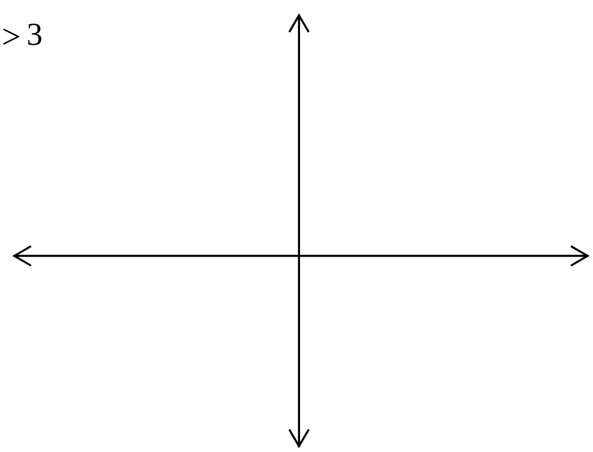


25. Algebraically determine whether the function is even, odd, or 25.____________________

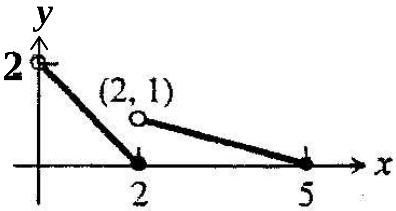
neither by stating and using their respective definitions: $y = \dfrac{x^3}{x^2 - 1}$

In Exercise 33, (a) draw the graph of the function labeling key points. 33b)D:_________________________
Then find its (b) domain and (c) range.

33. $f(x) = \begin{cases} 4 - x^2, & x < 1 \\ (3/2)x + 3/2, & 1 \le x \le 3 \\ x + 3, & x > 3 \end{cases}$ 33c)R:_________________________



43. Write a piecewise formula for the function below.



49. If $f(x) = x + 5$, $g(x) = x^2 - 3$ find each of the following and circle your answers. *Show all steps*!

(a) $f\left(g(x)\right)$

(b) $g\left(f(x)\right)$

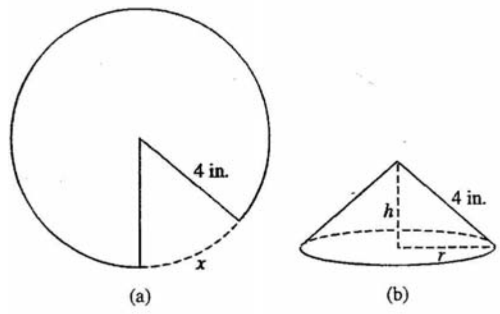
(c) $f\left(g(0)\right)$

(d) $g\left(f(0)\right)$

(e) $g\left(g(-2)\right)$

(f) $f\left(f(x)\right)$

65. Begin with a circular piece of paper with a 4-in. radius as shown in (a). Cut out a sector with an arc length of x. Join the two edges of the remaining portion to form a cone with radius r and height h, as shown in (b).



a) Explain why the circumference of the base of the cone is $8\pi - x$.

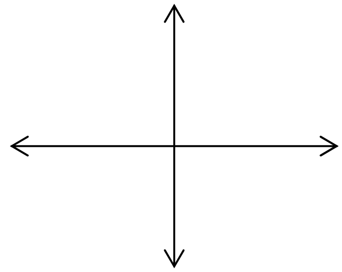
b) Express the radius r as a function of x.

c) Express the height h as a function of x.

d) Express the volume V of the cone as a function of x.

Section 1.3

9. Graph the function $y = 3e^{-x} - 2$ and indicate asymptote(s).



State its domain, range, and intercepts.

Domain:____

Range:____

Intercepts:____

13. Rewrite the exponential expression to have the indicated base: 13.____________________

$\left(\frac{1}{8}\right)^{2x}$, base 2

25. The half-life of phosphorus-32 is about 14 days. There are 6.6 grams present initially.

a. Express the amount of phosphorus-32 remaining as a function of time t. 25a.__________________________

b. When will there be 1 gram remaining? Solve algebraically. 25b.__________________________

31. Determine how much time is required for an investment to triple in value if interest is earned at the rate of 5.75% compounded daily? Solve algebraically. 31.__________________________

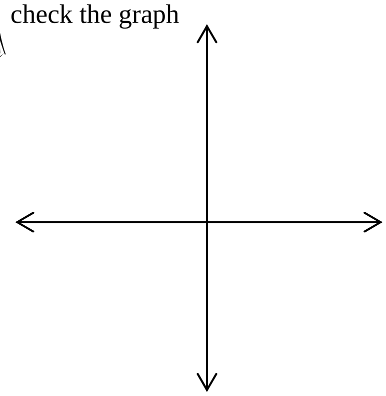
Section 1.4
Parametric equations are given below. (#11, 19)

Complete the table and sketch the curve represented by the parametric equations (label the initial and terminal points as well as indicate the direction of the curve). Then eliminate the parameter and write the corresponding rectangular equation whose graph represents the curve. Be sure to define the portion of the graph of the rectangular equation traced by the parametric equations.

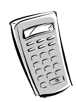
11. $x = 4\sin t, \quad y = 2\cos t, \quad 0 \le t \le 2\pi$

 check the graph

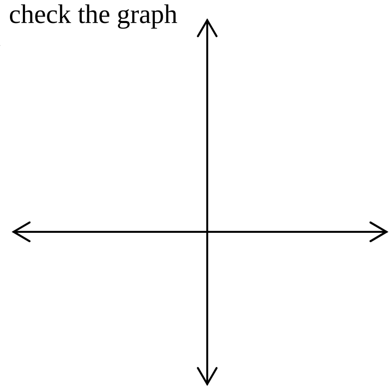
t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$3\pi/2$	2π
x							
y							



19. $x = 2t - 5, \quad y = 4t - 7, \quad -2 \le t \le 3$

 check the graph

t	-2	-1	0	1	2	3
x						
y						



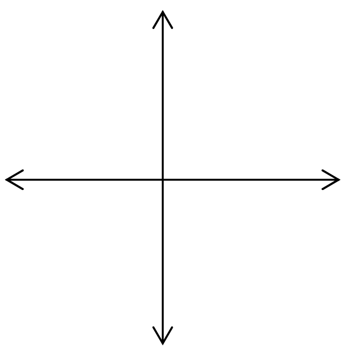
Section 1.5

7. Algebraically find the inverse of $y = \dfrac{3}{x-2} - 1$ 7.______________________

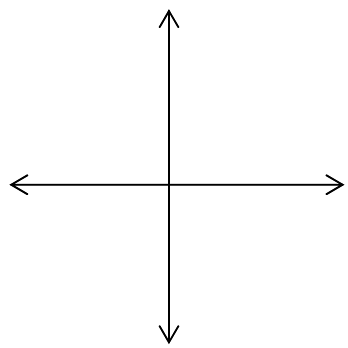
15. If $f(x) = x^3 - 1,$ find f^{-1} and verify that $f\left(f^{-1}(x)\right) = f^{-1}\left(f(x)\right) = x$

In exercises 33 & 35, draw the graph and determine the domain and range of the function.

33. $y = 2\ln(3-x) - 4$ D:________________ R:________________



35. $y = \log_2(x+1)$ D:________________ R:________________



In exercises 37 & 39, solve the equation algebraically. Support your solution graphically.

37. $(1.045)^t = 2$

39. $e^x + e^{-x} = 3$

In exercises 41-42, solve for *y* algebraically.

41. $\ln y = 2t + 4$

42. $\ln(y-1) - \ln 2 = x + \ln x$

Section 1.6

7. Give the measure of the angle in radians and degrees: $\sin^{-1}(0.5)$

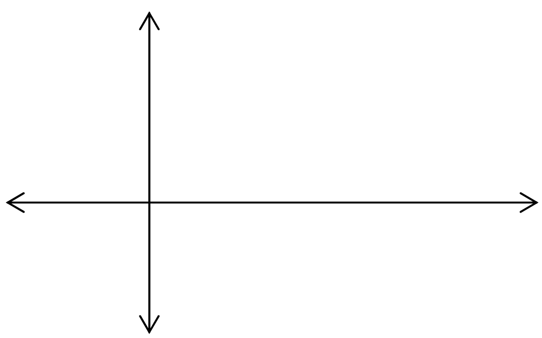
rad:____________

deg:____________

15. Without a calculator, sketch one cycle of the curve $y = -4\sin\dfrac{\pi}{3}x$ and identify and amplitude and period and label your axes.

Amp:___________

Period:_________

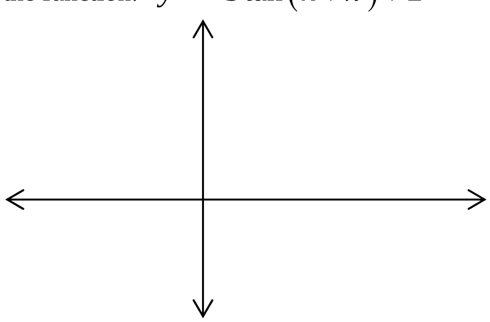


19. Determine (a) the period, (b) the domain, and (c) the range, and (d) sketch one cycle of the function: $y = -3\tan(x+\pi) + 2$

a) Period:_______________

b) D:____________________________

c) R:____________________________



Algebraically, solve each equation in the specified interval (leave answers in radians or π radians where applicable). (#25, 27, 29)

25. $\tan x = 2.5,\ \ 0 \le x < 2\pi$ 

25.________________________________

27. $\csc x = 2,\ \ \ 0 < x < 2\pi$

27.________________________________

29. $\cos x = -0.5,\ \ -\infty < x < \infty$

29.________________________________

31. Evaluate the expression in exact form: $\sin\left(\cos^{-1}\left(\frac{7}{11}\right)\right)$

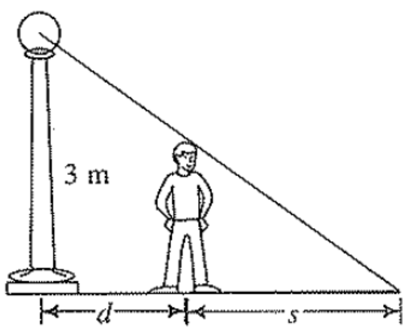
31.____________________

Forming Functions from Verbal Descriptions: Section 4.7 from Advanced Mathematics Precalculus Book (purple)

***** Use your**  **for all of the following applications *****

7. The height of a cylinder is twice the diameter. Express the total surface area A as a function of the height h.

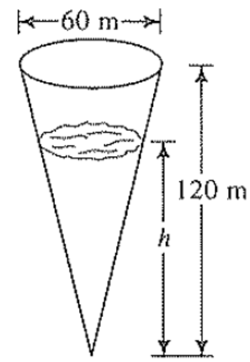
9. A light 3 m above the ground causes a boy 1.8 m tall to cast a shadow s meters long measured along the ground, as shown. Express s as a function of d, the boy's distance in meters from the light.



Ex. 9

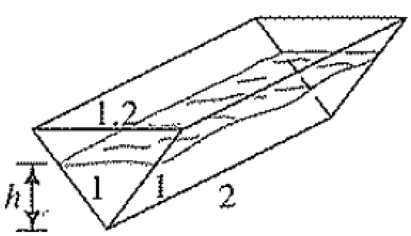
11. A box with a square base has a surface area (including the top) of 3 m^2. Express the volume V of the box as a function of the width w of the base.

19. Water is flowing at a rate of 5 m^3/s into the conical tank shown at the right.
a) Find the volume *V* of the water as a function of the water level *h*.
b) Find *h* as a function of the time *t* during which water has been flowing into the tank.



Ex. 19

20. A trough is 2 m long, and its ends are triangles with sides of length 1 m, 1 m, and 1.2 m as shown.
a) Find the volume *V* of the water in the trough as a function of the water level *h*.
b) If water is pumped into the empty trough at the rate of 6 L/min, find the water level *h* as a function of the time *t* after the pumping begins. (1 m^3 = 1000 L)



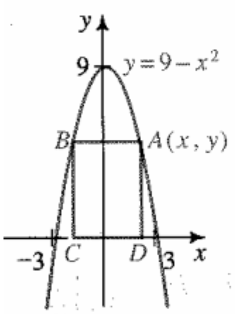
Ex. 20

23. As shown, rectangle *ABCD* has vertices *C* and *D* on the *x*-axis and vertices *A* and *B* on the part of the parabola $y = 9 - x^2$ that is above the *x*-axis.

a) Express the perimeter *P* of the rectangle as a function of the *x*-coordinate of *A*.

b) What is the domain of the perimeter function?

c) For what value of *x* is the perimeter a maximum?



Ex. 23

24. A sheet of metal is 60 cm wide and 10 m long. It is bent along its width to form a gutter with a cross section that is an isosceles trapezoid with 120° angles, as shown.

a) Express the volume *V* of the gutter as a function of *x*, the length in centimeters of one of the equal sides.

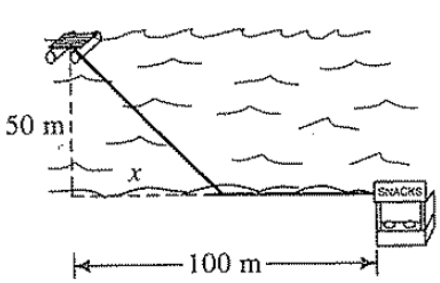
(*Hint:* Volume = area of trapezoid x length of gutter)

Cross section of gutter

25. From a raft 50 m offshore, a lifeguard wants to swim to shore and run to a snack bar 100 m down the beach, as shown at the left below.

a) If the lifeguard swims at 1 m/s and runs at 3 m/s, express the total swimming and running time *t* as a function of the distance *x* shown in the diagram.

b) Find the minimum time.



Ex. 25

Section 10.5- Polar

Replace the polar equation by an equivalent Cartesian equation.

25. $r^2 = 4r\sin\theta$

28. $r = \cot\theta\csc\theta$

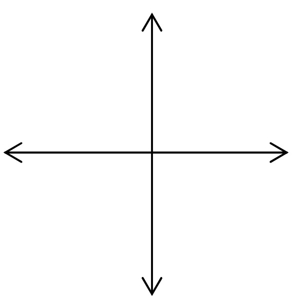
Replace the Cartesian equation by an equivalent polar equation.

40. $x - y = 3$

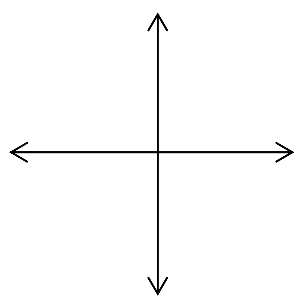
46. $x^2 + xy + y^2 = 1$ (solve for r^2)

Graph and name each polar curve.

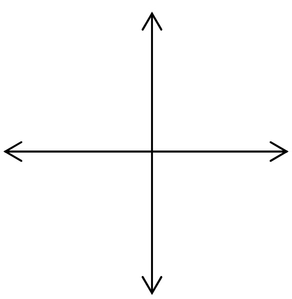
50. $r = 2 - 2\cos\theta$



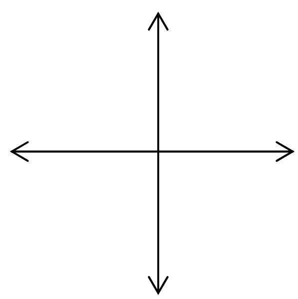
57. $r = 2\cos 3\theta$



58. $r = 1 + 2\sin\theta$



71. $r = 3\sin\theta$



QUICK REVIEW: (this is part of the summer assignment)

Complete the following blanks of extremely important concepts.

State the Pythagorean identities.

1. ____

2. ____

3. ____

State the double angle formulas.

6. $\sin 2x$ = ______________________________

7. $\cos 2x$ = ______________________________

____

____

State the sum and difference formulas.

4. $\cos(\alpha \pm \beta)$ = ______________________________

5. $\sin(\alpha \pm \beta)$ = ______________________________

State the limit definition of e.

8.______________________________________

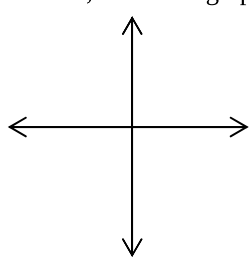
Find the simplest exact value of each of the following.

9. $\sin\frac{7\pi}{6}$ ___________ 10. $\cos\left(-\frac{\pi}{3}\right)$ _____________ 11. $\tan\frac{4\pi}{3}$ __________

12. $\csc\frac{5\pi}{4}$ __________ 13. $\sec\frac{5\pi}{6}$ _________________ 14. $\cot\frac{2\pi}{3}$ __________

15. For the function below, give the zeros (if none exist write *none*), domain, range, VA's, HA's, and/or points of discontinuity (holes- as ordered pairs) if any exist. Also, sketch its graph.

$f(x) = \frac{x+3}{2x^2+5x-3}$



zeros:________________

domain:______________________________

range:_______________________________

VA/HA/hole:___________________________

____

16. Simplify the expression: $\left(3x^{-1}+x^{3}\right)^{-2}$

16.__________________

17. Solve for *x*: $\log_5 x + \log_5(x-4) = 1$

17.__________________

ANSWERS to AP Calculus BC Summer Assignment Packet

SECTION 1.1

37. $y = -1$
43a. $k = 2$
43b. $k = -2$
51. $y = -\frac{3}{4}x + \frac{25}{4}$

SECTION 1.2

1. $A(d) = \frac{\pi d^2}{4}$

7a. $[1, \infty)$
7b. $[2, \infty)$
7c. check graph on calc

25. Odd

33a. check graph on calc
33b. All reals
33c. All reals

43. $f(x) = \begin{cases} 2 - x, & 0 < x \le 2 \\ \frac{5}{3} - \frac{x}{3}, & 2 < x \le 5 \end{cases}$

49a. $x^2 + 2$
49b. $x^2 + 10x + 22$
49c. 2
49d. 22
49e. -2
49f. $x + 10$

65a. because the circumference of the original circle was 8π and a piece of length x was removed

65b. $r = 4 - \frac{x}{2\pi}$

65c. $\frac{\sqrt{16\pi x - x^2}}{2\pi} = h$

65d. $V(x) = \frac{(8\pi - x)^2\sqrt{16\pi x - x^2}}{24\pi^2}$

SECTION 1.3

9. check graph on calc
Domain: All reals
Range: $(-2, \infty)$
x-intercept: ≈ 0.405
y-intercept: 1

13. 2^{-6x}

25a. $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$

25b. About 38.1145 days later

31. 19.108 years

SECTION 1.4

11. check graph on calc
Initial point and terminal point: (0, 2)
$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

19. check graph on calc
Initial point: $(-9, -15)$
Terminal point: (1, 5)
$y = 2x + 3;\ -9 \le x \le 1$

SECTION 1.5

7. $f^{-1}(x) = \frac{3}{x+1} + 2$

15. $f^{-1}(x) = (x+1)^{1/3}$ or $\sqrt[3]{x+1}$

33. D: $x < 3$, R: all reals, check graph on calc

35. D: $x > -1$, R: all reals, check graph on calc

37. $t = 15.75$

39. $x = \ x = \ln\left(\frac{3 \pm \sqrt{5}}{2}\right)$

41. $y = e^{2t+4}$

42. $y = 2xe^x + 1$

SECTION 1.6

7. $\frac{\pi}{6}$ radians, $30°$

15. A = 4, P = 6

19a. π

19b. all reals, $x \neq \frac{\pi}{2} + \pi n$

19c. all reals

19d. check graph on calc

25. x = 1.190, 4.332

27. $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$

29. $x = \frac{2\pi}{3} + 2\pi n, \quad x = \frac{4\pi}{3} + 2\pi n$

n is an integer

31. $\frac{6\sqrt{2}}{11}$

Forming Functions from Verbal Descriptions

7. $A(h) = \frac{5}{8}\pi h^2$

9. $s(d) = 1.5d$

11. $V(w) = \frac{3w - 2w^3}{4}$

19a. $V(h) = \frac{\pi}{48}h^3$

19b. $h(t) = \sqrt[3]{\frac{240t}{\pi}}$

20a. $V(h) = 1.5h^2$

20b. $h(t) = \sqrt{0.004t}$

23a. $P(x) = -2x^2 + 4x + 18$

23b. $0 < x < 3$

23c. 1

24a. $V(x) = 750\sqrt{3}\left(40x - x^2\right)$

24b. 20

25a. $t(x) = \sqrt{2500 + x^2} + \frac{1}{3}(100 - x)$

25b. about 80.5 sec

SECTION 10.5

25. $x^2 + (y-2)^2 = 4$

28. $y^2 = x$

40. $r = \frac{3}{\cos\theta - \sin\theta}$

46. $r^2 = \frac{1}{1 + \cos\theta\sin\theta}$

50, 57, 58, 71- check on your calculator

Quick Review

1. $\sin^2 x + \cos^2 x = 1$
2. $\tan^2 x + 1 = \sec^2 x$
3. $1 + \cot^2 x = \csc^2 x$
4. $\cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
5. $\sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
6. $2\sin x\cos x$
7. $\cos^2 x - \sin^2 x$, $1 - 2\sin^2 x$, $2\cos^2 x - 1$
8. $\lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x = e$
9. – ½
10. ½
11. $\sqrt{3}$
12. $-\sqrt{2}$
13. $-\frac{2\sqrt{3}}{3}$
14. $-\frac{\sqrt{3}}{3}$
15. zeros: none
    domain: all reals except x = – 3, ½
    range: all reals except y = 0
    hole at x = – 3
    VA at x = ½ , HA at y = 0
16. $\frac{x^2}{\left(3 + x^4\right)^2}$
17. x = 5